

# Multi-robot Allocation of Tasks with Temporal and Ordering Constraints

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## Abstract

Task allocation is ubiquitous in computer science and robotics, yet some problems have received limited attention in the computer science and AI community. Specifically, we will focus on multi-robot task allocation problems when tasks have time windows or ordering constraints. We will outline the main lines of research and open problems.

## Introduction

Many real-world problems have constraints on when, where, and in what order tasks need to be executed by robots. However, in the computer science and AI community limited attention has been devoted to allocation of tasks that have to be completed within a specified time window or have ordering or synchronization constraints. Other communities have studied time-window versions of many problems, such as Vehicle Routing Problem (VRP) with Time Windows, and Team Orienteering Problem (TOP) with Time Windows.

Time windows make task allocation difficult because the algorithms need to take into account both the spatial and the temporal relationships among the tasks (Kumar, Cirillo, and Koenig 2013). Dealing with general time windows in task allocation remains an open problem (Koenig, Keskinocak, and Tovey 2010).

The widely accepted taxonomy for multi-robot task allocation (MRTA) problems (Gerkey and Matarić 2004) classifies robots, tasks, and time as follows:

- *Single-task robots (ST) vs. multi-task robots (MT)*: ST robots can do at most one task at a time, MT robots can work on multiple tasks simultaneously.
- *Single-robot tasks (SR) vs. multi-robot tasks (MR)*: SR tasks require exactly one robot to be executed, while multiple robots are needed for MR tasks.
- *Instantaneous (IA) vs. time-extended (TA) assignments*: Tasks are allocated as they arrive in IA, and scheduled over a planning horizon in TA.

The taxonomy in (Korsah, Stentz, and Dias 2013) focuses on schedule dependencies in individual robot schedules and across robot schedules, but does not address different aspects of time-extended assignments, which is what we ad-

dress here. We call this class of problems with temporal and ordering constraints between tasks *MRTA/TOC*.

## MRTA/TOC: Multi-robot Task Allocation with Temporal and Ordering Constraints

We assume there is a finite set of robots and a set of tasks. A robot may have a location, speed, route, and/or schedule. A task may have a location, earliest start, latest finish time, expected duration, cost, demand, and reward.

Constraints on tasks can be in the form of time windows, which specify the period of time when a task can be done. Ordering constraints specify a dependency between pairs of tasks. They are usually represented as directed acyclic graphs, where each node in the graph represents a task, and each edge indicates a precedence constraint. In addition to precedence constraints, ordering constraints can be synchronization constraints, which specify, for instance, that two tasks have to start at the same time or that a task has to start a specific amount of time after another finishes.

We use the term *synchronization constraints* when there is a specific time involved and *precedence constraints* when there is only an ordering constraint. Those two types of constraints are indicated in the Operation Research literature as General Precedence Relationship (GPR), i.e., task  $j$  has to start within a given amount of time after the completion of task  $i$  or task  $j$  can start any time after the end of task  $i$  (Monma 1981). We prefer to use terms that make explicit the difference between those two types of constraints. More details in (Nunes et al. 2017).

The objective is to optimize some function of the cost (or reward) for doing the tasks for all the robots. Cost can be a temporal measure, such as the makespan – the time difference between the end of the last task and the start of the first task, a spatial measure, such as distance traveled, a combination of both, or some other measure.

## Connections with Other Problems

Multi-robot task allocation is similar to the vehicle routing problem (VRP) (Dantzig and Ramser 1959) the team orienteering problem (TOP) (Chao, Golden, and Wasil 1996). Variants of those problems include time windows, e.g., the vehicle routing problem with time windows (Kolen, Kan, and Trienekens 1987; Desrochers et al. 1988; Solomon and

Desrosiers 1988; Toth and Vigo 2002), or the team orienteering problem with time windows (Labadie et al. 2012; Rubinstein, Smith, and Barbulescu 2012; Coltin and Veloso 2014a)). Even though MRTA problems make different assumptions on the number of robots, task homogeneity, environment dynamics caused by failures or interference with other robots, and communication restrictions, there is much to be learned from work done in those other areas.

The solutions to several variants of VRP with time windows, such as multi-depot (Polacek et al. 2004; Kang, Lee, and Lee 2005), precedence and synchronization constrained (Bredström and Rönnqvist 2008; Korsah et al. 2012), have been extended to MRTA/TOC settings.

In MRTA/TOC problems communication is important and often constrained. Some approaches, such as (Nunes and Gini 2015), cut down on communication costs by using auctions for allocation and having each robot keep its own schedule.

In the team orienteering problem with time windows, an origin and destination pair is given. The goal is to search for control points to visit between the origin and destination such that the profit (or score function) is maximized while respecting all the constraints. Each control point is associated with a profit (or score), and each edge connecting control points is weighted by the cost of moving between the control points (Labadie et al. 2012). Control points are equivalent to tasks in MRTA/TOC. One application of team orienteering problems, dial-a-ride, has gained popularity in MRTA/TOC (Coltin and Veloso 2014a). In dial-a-ride, the problems are over-constrained, which means that not all the tasks can be performed, and thus the goal is to find the subset of tasks that maximizes the total profit (Rubinstein, Smith, and Barbulescu 2012).

## Temporal Models

### Relationships between time intervals

Time can be modeled using points (e.g., 9:00am) or intervals (e.g., [9:00-13:00]). The interval representation (Allen 1983) uses a set of relationships that hold between pairs of time intervals. The relationships can be used to model partial or complete ordering constraints between tasks, – for example, task  $X$  should be done before, after, or at the same time as task  $Y$ . The “ $X$  before  $Y$ ” operator can be used to describe precedence constraints between tasks, while the “ $X$  equal  $Y$ ” operator describes a synchronization constraint between the start and end points of two tasks.

### Simple Temporal Networks (STN)

Dechter (Dechter, Meiri, and Pearl 1991) proposed to represent a class of temporal constraints with a graph, called a simple temporal network (STN). Nodes represent time point variables or time events, and weighted edges represent inequality constraints between time points. To reduce computational complexity, this representation requires exactly one constraint between pairs of time point variables. A solution to the scheduling problem can be computed in polynomial time using the Floyd-Warshall algorithm. In an STN the re-

lationship between time windows can be represented by establishing constraints between start and finish times of tasks.

STNs are used in MRTA problems (Barbulescu et al. 2010; Gombolay, Wilcox, and Shah 2013; Nunes and Gini 2015) because constraint consistency can be efficiently verified in polynomial time (Dechter, Meiri, and Pearl 1991; Xu and Choueiry 2003; Planken, de Weerd, and van der Krogt 2008). An important feature of STNs is that new time points and constraints can be dynamically added in polynomial time (Cesta and Oddi 1996), which is beneficial in dynamic domains where new tasks can appear and disappear. STNs have been extended to multi-agent settings (Boerkoel and Planken 2012) and to scenarios with uncertainties (Vidal 1999; Tsamardinos 2002; Fang, Yu, and Williams 2014).

### Time Window Constraints

A time window specifies a temporal interval constraint on the start and finish time of a task. A time window has a lower bound value, usually the task’s earliest start time, and an upper bound value, usually the task’s latest finish time. A task can also have a latest start time and an earliest finish time, resulting in a time window of the form [earliest start time, latest start time, earliest finish time, latest finish time]. This representation implicitly provides an upper bound to the task duration. When the earliest and latest start times are the same, the time window specifies only a start time. Same for finish time. If only a start time is given the finish time is assumed to be the end of the scheduling horizon; similarly if only a finish time is given the start time is the beginning of the scheduling horizon.

Specifying earliest and latest start or finish times increases the flexibility of task allocation, but increases the search space because there are multiple ways of scheduling a task within its time window. The use of time windows for auction-based task allocation to agents was pioneered in the MAGNET system, which proposed various task allocation algorithms (Collins and Gini 2006).

Time windows can be used to model many types of temporal relationships among tasks. For instance, deadline constraints (Amador, Okamoto, and Zivan 2014; Luo, Chakraborty, and Sycara 2015) impose constraints on the latest time robots can arrive to tasks before the task expires.

Task allocation problems with time windows are generally (except for a few special cases, e.g., (Melvin et al. 2007)) NP-hard (Solomon and Desrosiers 1988), and finding a feasible solution is NP-complete (Savelsbergh 1985). The inclusion of time windows makes it harder to design efficient approximation algorithms.

### Precedence and Synchronization Constraints

Precedence constraints specify a partial or total order for the tasks, without necessarily providing a specific time window for each task. Time windows can be used to specify implicitly precedence or synchronization constraints, but in general they are not sufficient. Two time windows with the same start time do not necessarily indicate a synchronization constraint. Time windows that overlap are not sufficient to specify precedence constraints.

Solutions to MRTA/TOC problems might assign to different robots tasks that depend on each other. This creates cross-schedule dependencies among robots (Jones, Dias, and Stentz 2011; Korsah et al. 2012), which are undesirable because exogenous events during execution affecting one robot will also affect the robots that depend on it.

Precedence and synchronization constraints impose partial ordering between tasks, which can be used to eliminate candidate solutions that violate the ordering. Instead tasks with time windows are independent of each other and can be done in any order, as long as there is enough time to reach the tasks and execute them within their time windows.

In (Luo 2014) a model for tasks with set precedence constraints is presented. The model divides tasks into disjoint sets with strict ordering between the sets, and assumes that each robot can do at most one task per set. The model heavily constrains the type of allowable precedence graphs, but the algorithm proposed is proved to be sound and complete. A very general model for allocation of tasks with any type of precedence constraint is presented in (McIntire, Nunes, and Gini 2016).

### Optimization Objectives

Applications of MRTA/TOC problems require the robots to achieve a given optimization objective. There can be a single or multiple objectives. Optimization objectives might require a quantity to be minimized, usually a cost (Chopra and Egerstedt 2012; Gombolay, Wilcox, and Shah 2013; Nunes and Gini 2015) or regret (Heap and Pagnucco 2014), or to be maximized, usually a score (Ponda et al. 2010) or a reward (Koes, Nourbakhsh, and Sycara 2005; Melvin et al. 2007; Korsah et al. 2012).

Common optimization objectives for MRTA/TOC problems include:

- MiniSUM, i.e. minimize the sum of the robot path costs over all the robots (Lagoudakis et al. 2005; Coltin and Veloso 2014b; Chopra and Egerstedt 2012) or some time measure over robot paths (Barbulescu et al. 2010; Heap and Pagnucco 2014)).
- MiniMAX, i.e. minimize the maximum path cost of a robot over all the robots (Lagoudakis et al. 2005). Instead of minimizing the maximum path cost, a similar objective function is to minimize the makespan, i.e. the time difference between the start of the first and the end of the last task (Graham et al. 1979; Nunes and Gini 2015).
- MiniAVE: i.e. minimize the average per task cost of the path over all the tasks. The per task cost is the cost of the path from the initial location of the robot to the task location (Lagoudakis et al. 2005). This is known as the Traveling Repairman Problem (Fakcharoenphol, Harrelson, and Rao 2007), where the objective is to minimize the wait time of the customers for a repairman.
- Minimize lateness or tardiness, which is the difference between the earliest start time of a task and the actual arrival time of the robot (Ponda et al. 2010; Rubinstein, Smith, and Barbulescu 2012).
- Maximize the number of tasks completed (Lau, Sim, and Teo 2003) or minimize the number of tasks missed.

- Minimize the number of robots used. This is common in vehicle routing problems, where the number of vehicles available is unlimited (Luo and Schonfeld 2007; Bräysy and Gendreau 2005a; Desrochers et al. 1988).
- Maximize profit, measured as the difference between the reward of tasks and their respective costs (Melvin et al. 2007; Korsah et al. 2012), or as the team utility (Koes, Nourbakhsh, and Sycara 2005; Ponda et al. 2010; Amador, Okamoto, and Zivan 2014).

Multi-objective problems are common, especially when objectives are combined through linear aggregation. For example, makespan and distance are minimized in (Ponda et al. 2010; Nunes and Gini 2015), while (Gombolay, Wilcox, and Shah 2013) also minimizes workspace overlap. In (Alighanbary, Kuwata, and How 2003) a multi-objective function minimizes the maximum and average task completion times, as well as total idle times.

### Dynamic Task Release and Execution

Execution of tasks in MRTA/TOC problems vary according to the dynamics considered. Dynamics may be due to faulty robots, changes in estimated cost due to uncertainties, changes in task definitions, online arrival of tasks, addition of robots to the team, and other changes made by external agents (Sariel-Talay, Balch, and Erdogan 2009). While the execution aspect is outside the task allocation scope, we consider dynamics caused by task arrival and during task execution separately.

Some dynamics are caused by the arrival of tasks over time without further knowledge of future tasks. Usually when a new task arrives there is already an existing allocation for previously scheduled tasks that have not yet been performed. Thus, replanning occurs at task arrivals, while robots are executing previously assigned tasks (Cordeau and Laporte 2007). In (Nunes and Gini 2015) both deterministic and dynamic task arrivals are considered, assuming the robots know the map where tasks appear. In contrast, problems defined as online pickup and delivery problems or dial-a-ride include not only online arrival of tasks but other uncertain events, such as vehicle breakdowns and delays (Cordeau and Laporte 2007). Recent examples of online pickup and delivery consider transfers in addition to the arrival of tasks with hard temporal constraints (Coltin and Veloso 2014b; 2014a).

The dynamics that occur during plan execution (Sariel-Talay, Balch, and Erdogan 2009; Shah, Conrad, and Williams 2009) are very important for the practical use of robots, because execution can fail due to many reasons and replanning is essential to maintain some level of efficiency. In (Barbulescu et al. 2010) dynamics during execution are created by unexpected events and changes in costs and constraints; in (Ponda et al. 2010) dynamics are caused by breaks in communication links, which may cause conflicting assignments, as more than one robot could be assigned the same task.

## Typical Solution Approaches

### Centralized Solutions

Centralized methods rely on a central controller that allocates tasks to robots. The robots simply execute the assigned schedule. Optimal solutions are typically computed using Branch-and-Bound (Clausen 1997). Branch-and-Bound searches the state space of candidate solutions represented as a tree and uses upper and lower bounds of the optimal solution to prune the branches of the search tree that have costs higher than the computed lower bounds. Variants of Branch-and-Bound, such as Branch-and-Cut (Bard, Kontoravdis, and Yu 2002; Ropke, Cordeau, and Laporte 2007), Branch-and-Price (Dohn, Kolind, and Clausen 2009; Korsah et al. 2012), and Branch-Price-and-Cut (Barnhart, Hane, and Vance 2000; Desaulniers 2010; Archetti, Bouchard, and Desaulniers 2011) have been used for VRP with time windows problem, but not as much for MRTA/TOC problems.

MRTA/TOC is intractable for large numbers of robots and tasks. Thus, the focus of MRTA/TOC solutions is largely on approximation and heuristic solution methods. For example, a Mixed Integer Linear Programming solver is used in conjunction with a task sequencer to solve separately task allocation and task sequencing efficiently. The method does not provide theoretical guarantees, but experimentally produces near optimal schedules for up to 10 robots and 500 tasks in less than 20 seconds (Gombolay, Wilcox, and Shah 2013).

Another way to gain computational efficiency is to use metaheuristic approaches. Metaheuristics are algorithmic templates that use heuristics to produce approximate solutions for hard combinatorial optimization problems. Unlike other combinatorial optimization algorithms, metaheuristics may allow lower quality solutions in the search process to escape local optima (Bräysy and Gendreau 2005b; Vidal et al. 2013). Metaheuristic approaches have been shown to outperform other methods (e.g. construction heuristics and local search) for standard benchmarks for VRP and TOP with time windows (Bräysy and Gendreau 2005b; Hu and Lim 2014). Recent trends in the metaheuristic literature seek to reduce the computation time and improve the solution quality by using parallelization and hybridization of different heuristics and exact techniques (e.g., (Mitiche, Bougaci, and Gini 2015)). However, metaheuristic parameters remain hard to tune (Bräysy and Gendreau 2005b; Birattari 2009; Mitiche, Godoy, and Gini 2015).

In addition to scalability issues, centralised methods suffer from being a single point of failure. In addition, they have to generate a new solution whenever new tasks appear during execution or when significant delays disrupt the execution schedule. This is not only undesirable computationally, but it tends to produce instability since tasks might be re-located in the new solution. To the contrary, distributed allocation methods degrade gracefully in the presence of communication interruptions, communication errors, and robot malfunctions.

### Decentralized Solutions

Decentralized approaches vary widely. Here we focus on (1) distributed constraint optimization and (2) market-based al-

gorithms since these have received a great deal of attention in the MRTA community.

**Distributed Constraint (DCOP)-Based Methods.** MRTA/TOC problems can be modeled as a Distributed Constraint Optimization Problem (DCOP) (Maheswaran et al. 2004) and solved using DCOP methods. Solving DCOP exactly is NP-hard and impractical even for unconstrained MRTA problems (Junges and Bazzan 2008). Optimal solutions also require an exponential coordination overhead (i.e. communication computation). Thus, approximate methods such as Max-Sum have been used for task allocation in sensor networks (Farinelli, Rogers, and Jennings 2014) and in RoboCup Rescue (Ramchurn et al. 2010; Pujol-Gonzalez et al. 2015; Parker, Farinelli, and Gini 2017).

In (Ramchurn et al. 2010) the Fast Max-Sum algorithm is proposed and shown to be robust in situations where the number of tasks is dynamic; the approach reduced the computation time, number and size of messages sent compared to Max-Sum, but it is still exponential. When the constraints are expressed using Tractable Higher Order Potentials the computation time can be reduced to polynomial (Pujol-Gonzalez et al. 2015).

Another approximation method is LA-DCOP (Scerri et al. 2005; Farinelli et al. 2006), which uses token passing to pass the token to a randomly chosen agent. This tends to guide the search quickly towards a greedy solution.

**Market Based Methods.** Among the decentralized algorithms, sequential auction- and negotiation-based algorithms (Sariel-Talay, Balch, and Erdogan 2009; Nanjanath and Gini 2010; Ponda et al. 2010; Heap and Pagnucco 2014; Nunes and Gini 2015) are widely used. Sequential auction algorithms produce solutions that are two away from optimal in the worst-case in both single-item (Lagoudakis et al. 2004) and multi-item auctions (Choi, Brunet, and How 2009). This, together with the ease of implementation and extension to dynamic scenarios and robust execution (Nanjanath and Gini 2010) makes single-item sequential auctions an attractive solution. However, the greedy nature of sequential auctions and the complex structure of most MRTA/TOC problems cause the addition of temporal constraints to auction algorithms to produce suboptimal solutions (Nunes, Nanjanath, and Gini 2012). Temporal modeling and balancing between temporal- and distance-based objectives can help auctions perform better (Ponda et al. 2010; Nunes and Gini 2015).

Auctions distribute the computation to individual agents but require communication to share bids and results. To reduce the need for communication, several approaches use consensus algorithms (Zavlanos, Spesivtsev, and Pappas 2008; Choi, Brunet, and How 2009; Ponda et al. 2010), where each agent determines independently which tasks it should do. An equilibrium is reached by iteratively sharing information with neighbors and re-allocating tasks if needed. In (Godoy and Gini 2012) the Consensus Based Bundle Algorithm (CBBA) (Choi, Brunet, and How 2009) is extended to optimize the number of completed tasks.

Despite the development of many decentralized methods

for MRTA/TOC problems, very limited work offers theoretical analysis of the quality of these solutions. There is a need for theoretical performance bounds for both centralized and decentralized heuristics for the MRTA/TOC problem.

Swarm-based approaches have been proposed for various tasks, such as foraging, where robots need to find food and bring it to the nest (Lerman et al. 2006; Brutschy et al. 2014) or where swarms of robots are allocated different monitoring tasks without any communication among the robots (Berman et al. 2009). Swarm methods often work well but do not have theoretical guarantees.

## Open Issues and Future Research

There are several open issues that need to be addressed, such as: (1) theoretical guarantees for approximate solutions, (2) richer and more complex temporal models with provably good and efficient algorithms, (3) models and algorithms for stochastic MRTA/TOC problems, (4) models and algorithms for MRTA/TOC problems that require multiple robots to work together on a task as well as for robots that can do more than one task at once.

Research in stochastic MRTA/TOC problems is still very sparse. The development of MRTA methods that take advantage of simulation and stochastic models to better plan under uncertainty is worth pursuing because robots often operate in uncertain environments.

There is also a need for work on theoretical guarantees for heuristic schedulers for MRTA/TOC problems. The NP-complete nature of the problem and the need for relatively fast planners has generated many heuristics. However, heuristics typically lack performance guarantees, which can be crucial for safety critical systems.

More complex temporal constraint types, such as disjunctive temporal models, need to be addressed, as well as combinations of precedence with synchronization constraints. A mix of these constraints might produce more expressive models for a larger set of real-world problems.

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