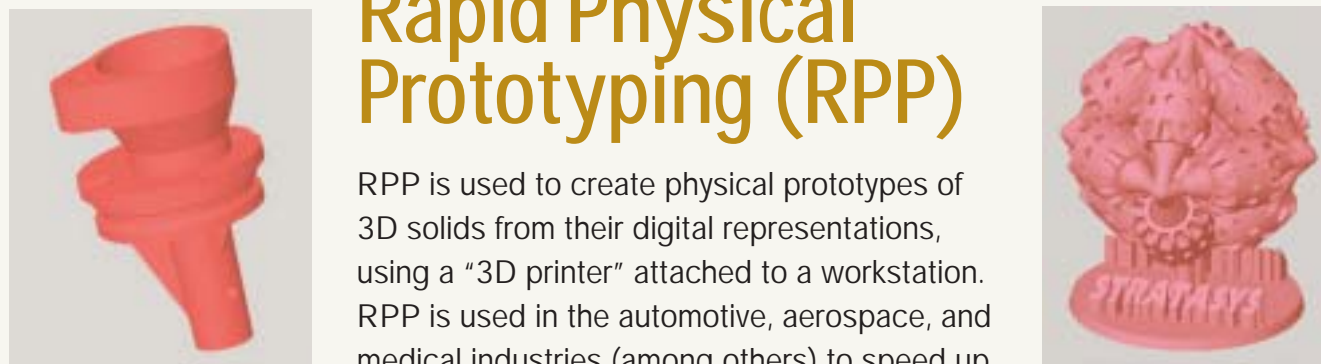# GEOMETRIC ALGORITHMS FOR RAPID PHYSICAL PROTOTYPING

Ravi Janardan
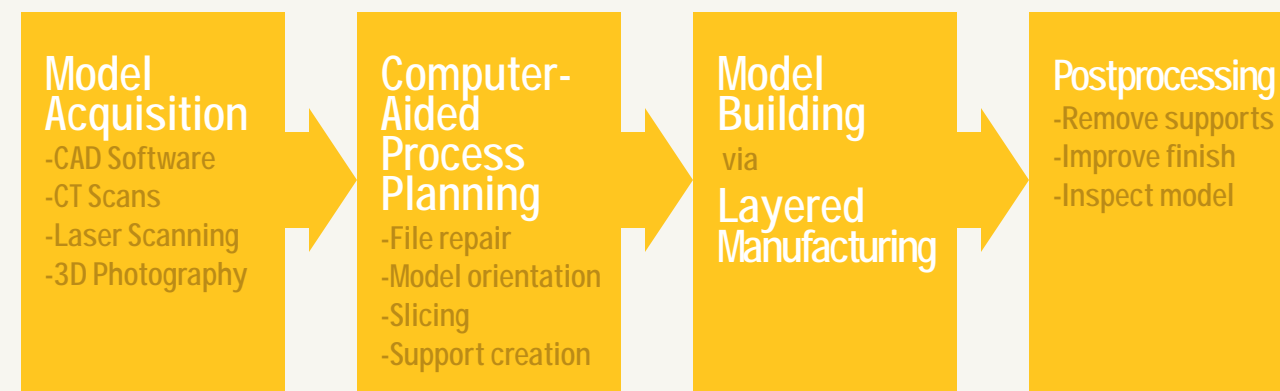
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Representative Publications

I. Ilinkin, R. Janardan, J. Majhi, J. Schwerdt, M. Smid, R. Sriram. A decomposition-based approach to layered manufacturing. Computational Geometry: Theory and Applications. (In press.)

M. Hon, R. Janardan, J. Schwerdt, M. Smid. Computing optimal hatching directions in layered manufacturing. Computer-Aided Design. (In press.)

J. Majhi, R. Janardan, J. Schwerdt, M. Smid. Multi-criteria geometric optimization problems in layered manufacturing. International Journal of Mathematical Algorithms. (In press.)

J. Schwerdt, M. Smid, R. Janardan, E. Johnson, J. Majhi. Protecting critical facets in layered manufacturing. Computational Geometry: Theory and Applications, 16:187-210, 2000.

J. Schwerdt, M. Smid, R. Janardan, E. Johnson. Protecting critical facets in layered manufacturing: Implementation and experimental results. Proc. 2nd Workshop on Algorithm Engineering and Experiments (ALENEX00), San Francisco, CA, Jan. 2000, pp. 43-58.

J. Majhi, R. Janardan, M. Smid, P. Gupta. On some geometric optimization problems in layered manufacturing. Computational Geometry: Theory and Applications, 12:219-239, 1999.

J. Majhi, R. Janardan, J. Schwerdt, M. Smid, P. Gupta. Minimizing support structures and trapped area in two-dimensional layered manufacturing. Computational Geometry: Theory and Applications,12:241-267, 1999.

J. Schwerdt, M. Smid, J. Majhi, R. Janardan. Computing the width of a three-dimensional point-set: An experimental study. ACM Journal of Experimental Algorithms, 4 article 8, 1999. (Published electronically at http://www.jea.acm.org/1999/SchwerdtWidth/.) Invited paper.
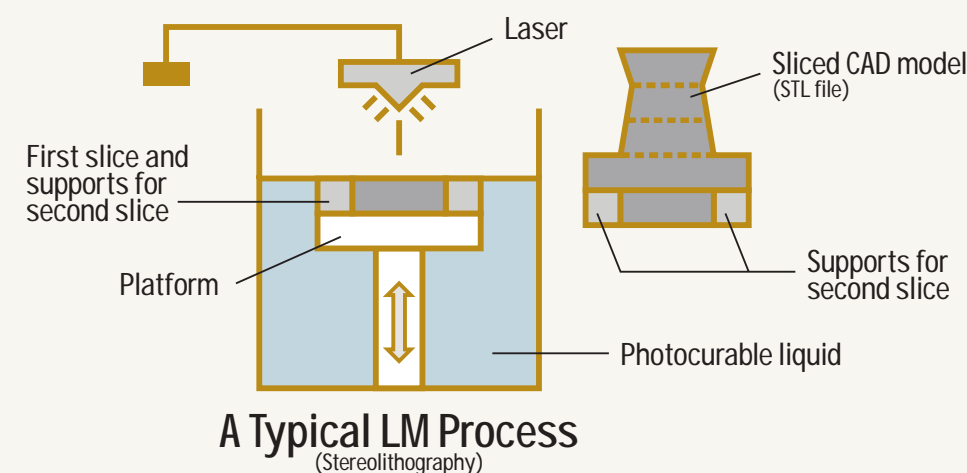
Rapid Physical Prototyping (RPP)

RPP is used to create physical prototypes of 3D solids from their digital representations, using a "3D printer" attached to a workstation. RPP is used in the automotive, aerospace, and medical industries (among others) to speed up the design cycle.

Model Acquisition
-CAD Software
-CT Scans
-Laser Scanning
-3D Photography

Computer-Aided Process Planning
-File repair
-Model orientation
-Slicing
-Support creation

Model Building via **Layered Manufacturing**

Postprocessing
-Remove supports
-Improve finish
-Inspect model

Lan or Internet

Layered Manufacturing (LM)

• Builds 3D model as a stack of 2D layers
• The virtual 3D model is oriented and sliced into parallel 2D layers
• Each layer is "printed" by having a laser trace the layer geometry on the surface of a light-sensitive liquid which then solidifies (e.g., 3D Systems' Stereolithography), or by depositing fine strands of molten plastic (e.g., Stratasys' FDM), etc.
• The layers stack up on a platform which moves down after each layer is built.
• Support structures are generated to prop up overhangs. These are precomputed and built along with the model.

Laser
Sliced CAD model (STL file)
First slice and supports for second slice
Supports for second slice
Platform
Photocurable liquid
A Typical LM Process (Stereolithography)

Geometric Considerations

The speed, accuracy, and cost of LM are affected by several factors that are purely geometric in nature.
In this project, we have designed efficient geometric algorithms to:
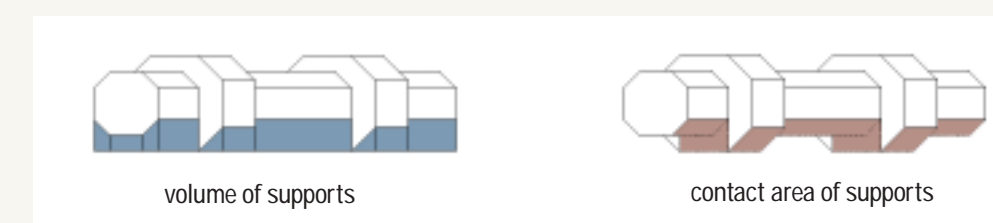• Compute model orientations that minimize the number of layers, support volume, and surface error ("jaggies").
• Decompose the model into smaller pieces for faster build times and reduced support requirements.
• Generate a compact description of support structures.
• Compute optimal paths for the printing tool (e.g., the laser) as it traces the layer geometry.
• Compute orientations that protect critical surfaces of the model from coming in contact with supports.

Sampling of Geometric Techniques Used

• Spherical Geometry
• Voronoi Diagrams, Arrangements
• Static and Dynamic Convex Hulls
• Cylindrical and Vertical Decomposition
• Geometric Duality
• Space sweep, Rotating Calipers, Ray-shooting
• Discrete and Continuous Optimization

Optimizing Surface Finish

Objective: The discretization of the model into layers creates stair-step-like artifacts on the physical prototype, whose severity depends on the chosen orientation. The goal is to compute an orientation which minimizes such artifacts.

Ideal solid (viewed end-on)
Stair-step height
Actual Prototype built (figure exaggerated for clarity)
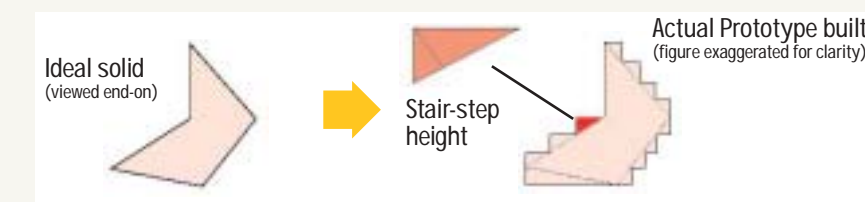
Results: Developed fast algorithm to minimize the maximum stair-step height. Algorithm needs only local (facet-level) geometry and uses spherical convex hulls (or spherical Voronoi diagrams).

Map facet normals to direction sphere
Compute optimal orientation from spherical convex hull

Support Optimization

Objective: Compute an orientation of the model which minimizes the volume of supports or the area of contact between supports and the model. (This improves the speed, cost, and accuracy of the process.)

volume of supports
contact area of supports

Results: Designed efficient algorithms to minimize support volume and contact-area for 3D convex and 2D non-convex models. Algorithms map the model to an arrangement on the sphere of directions, which partitions the sphere into cells where the solution is (essentially) invariant. Cells are searched to compute the optimal orientation.

map model to direction sphere
Arrangement and cell
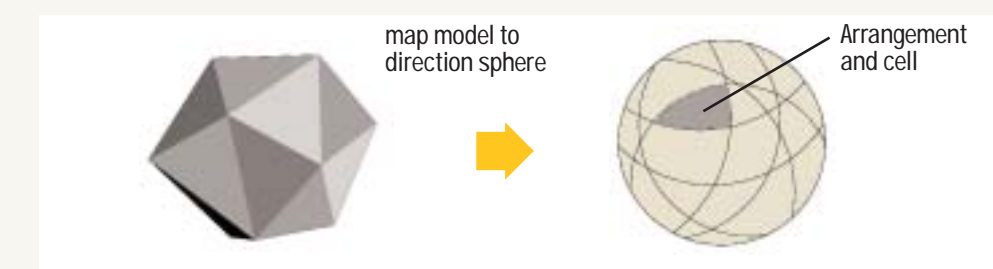
Decomposition-Based Approach to Layered Manufacuring

Objective: Compute a decomposition of the model which minimizes support requirements.

model w/o decomposition (needs supports)
decomposed model (no supports needed)
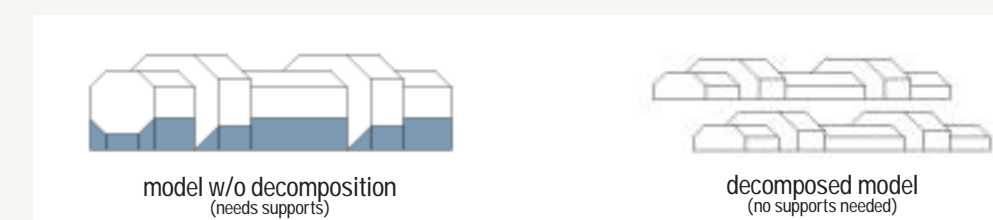
Results: Designed efficient decomposition algorithms that achieve significant support reduction. Algorithms based on sweeping and cylindrical decomposition.
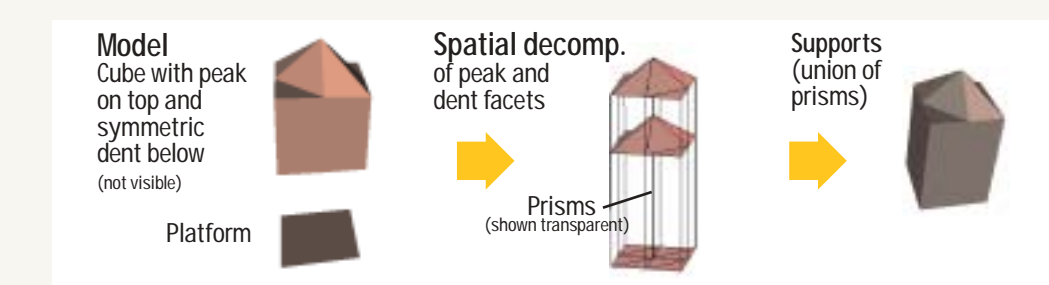
82% savings
30%
86%
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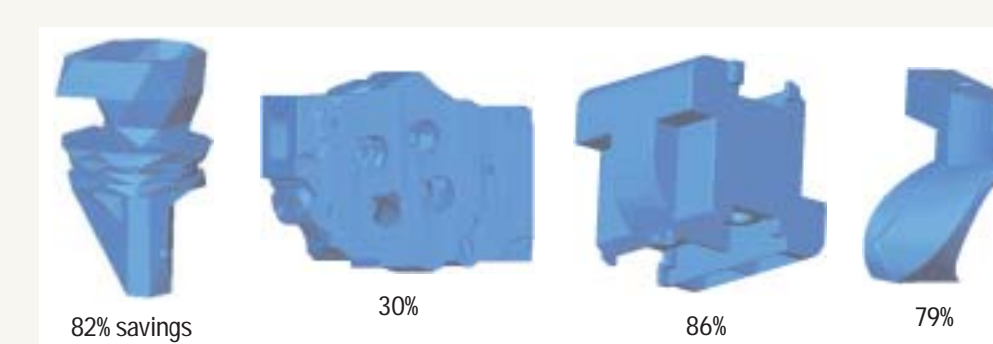
Support Computation Algorithms

Objective: Given a model and an orientation, compute a compact (combinatorial) description of the supports needed, as a collection of disjoint polyhedra.

model + supports
supports as polyhedra
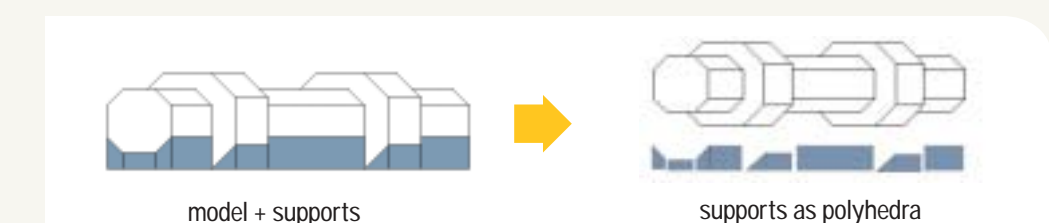
Results: Designed efficient algorithms that use vertical and cylindrical decomposition techniques to partition the space "outside" the model into a collection of prism-like polyhedra that constitute the supports.

Model
Cube with peak on top and symmetric dent below (not visible)
Platform
Spatial decomp.
of peak and dent facets
Prisms (shown transparent)
Supports (union of prisms)

Tool-Path Generation

Objective: Compute a direction along which the tool can fill in a layer so that the number of times the tool starts, stops, and changes direction is minimized. (This increases process speed and tool life.)

Typical Layer Many direction changes
Simple example
10 dir. changes
24 dir. changes
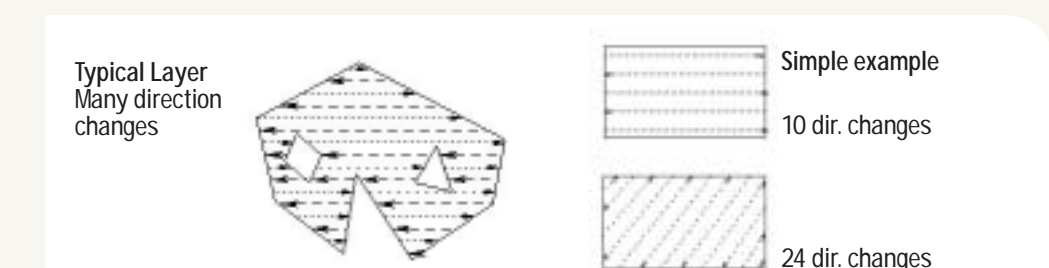
Results: Designed very efficient heuristics for both single layers and for all layers of the model. Based on a projection minimization method.

Fan Blade
Single layer
All layers
434 dir. changes (opt. = 426) Time: 0.01s (Sun UltraSparc10)
515 layers Time: 2.3s