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Department of Computer Science
and Engineering
University of Minnesota
4-192 Keller Hall
200 Union Street SE
Minneapolis, MN 55455-0159 USA

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MESH: A Flexible Distributed Hypergraph Processing System

Benjamin Heintz, Shivangi Singh, Rankyung Hong, Guarav Khandelwal, Corey Tesdahl, Abhishek Chandra

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MESH: A Flexible Distributed Hypergraph Processing System

Benjamin Heintz  
University of Minnesota  
Minneapolis, MN  
heintz@cs.umn.edu

Shivangi Singh  
University of Minnesota  
Minneapolis, MN  
singh486@umn.edu

Rankyung Hong  
University of Minnesota  
Minneapolis, MN  
hongx293@umn.edu

Gaurav Khandelwal  
University of Minnesota  
Minneapolis, MN  
khand052@umn.edu

Corey Tesdahl  
University of Minnesota  
Minneapolis, MN  
tesd0005@umn.edu

Abhishek Chandra  
University of Minnesota  
Minneapolis, MN  
chandra@umn.edu

Abstract—With the rapid growth of large online social networks, the ability to analyze large-scale social structure and behavior has become critically important, and this has led to the development of several scalable graph processing systems. In reality, however, social interaction takes place not just between pairs of individuals as in the graph model, but rather in the context of group interactions. It has been shown that many natural phenomena can be better modeled through a hypergraph model, resulting in the need to build scalable hypergraph processing systems. In this paper, we present MESH, a flexible distributed framework for scalable hypergraph processing. MESH provides an easy-to-use and expressive application programming interface that naturally extends the “think like a vertex” model common to many popular graph processing systems. Our framework provides a flexible implementation based on an underlying graph processing system, and enables different design choices for the key implementation issues of hypergraph representation and partitioning. We implement MESH on top of the popular GraphX graph processing framework in Apache Spark. Using a variety of real datasets, we experimentally demonstrate that MESH provides flexibility based on data and application characteristics, examine its scaling with cluster size, and show that it is competitive in performance to HyperX, another hypergraph processing system based on Spark, showing that simplicity and flexibility need not come at the cost of performance.

I. INTRODUCTION

The advent of online social networks and communities such as Facebook and Twitter has led to unprecedented growth in user interactions (such as “likes”, comments, photo sharing, and tweets), and collaborative activities (such as document editing and shared quests in multi-player games). This has resulted in massive amounts of rich data that can be analyzed to better understand user behavior, information flow, and social dynamics. The traditional way to study social networks is by modeling them as graphs, where each vertex represents an entity (e.g., a user) and each edge represents the relation or interaction between two entities (e.g., friendship). Myriad graph analytics frameworks [1]–[3] have been introduced to scale out the computation on massive graphs comprising millions or billions of vertices and edges.

While graph analytics has enabled a better understanding of social interactions between individuals, there is a growing interest [4] in studying groups of individuals as entities on their own. A group is an underlying basis for many social interactions and collaborations, such as users on Facebook commenting on an event of common interest, or a team of programmers collaborating on a software project. In these cases, individuals interact in the context of the overall group, and not simply in pairs. Further, the dynamics of many such systems may also be driven through group-level events, such as users joining or leaving groups, or finding others based on group characteristics (e.g., common interest).

Since such group-based phenomena involve multi-user interactions, it has been shown that many natural phenomena can be better modeled using hypergraphs than by using graphs [5]. Formally, a hypergraph is a generalization of a graph and is defined as a tuple \( H = (V, E) \), where \( V \) is the set of entities, called vertices, in the network, and \( E \) is the set of subsets of \( V \), called hyperedges, representing relations between one or more entities [6] (as opposed to exactly two in a graph).

Figure 1 illustrates the difference between a graph and a hypergraph. This figure shows a 5-vertex network, consisting of four groups (\( \{v_1, v_2\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_3, v_4\} \)). As can be seen from the figure, a graph can only capture binary relations (e.g., \( \{v_1, v_2\}, \{v_3, v_4\} \), etc.), some of which may correspond to distinct overlapping groups (e.g., \( \{v_3, v_4\} \) belongs to two distinct groups). On the other hand, a hypergraph can model all the groups unambiguously compared to a graph.

Recent work [7] has shown that hypergraph models can achieve a significant improvement in modeling accuracy compared to graph-based models for social interactions. While hypergraph algorithms have received much less attention than graph algorithms, there has been work on developing hypergraph counterparts for problems such as centrality estimation [8], shortest path computation [9], and others. These algo-

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1In this paper, we use “graph” to refer to a traditional dyadic graph.
A hypergraph can model groups unambiguously based on a graph processing framework. We use our system to implement, and can provide flexibility in terms of design choices. In this paper, we take this approach, and explore two key challenges in implementing a hypergraph processing system on top of a graph processing system: how to represent the hypergraph and how to partition this representation to allow efficient distributed computation. For our implementation, we choose the GraphX framework [1] in Apache Spark [14], though we expect our ideas to be applicable or extensible to other graph processing frameworks as well.

A. Research Contributions

- We present MESH, a distributed hypergraph processing system designed for scalable hypergraph processing, based on a graph processing framework.
- We present an expressive API for hypergraph processing, which extends the popular “think like a vertex” programming model [2] by treating hyperedges as first-class computational objects with their own state and behavior.
- We explore the impact of two key design questions in building a hypergraph processing system: how to represent the hypergraph and how to partition this representation for distributed computation. We examine multiple design choices and compare them both qualitatively and quantitatively.
- We implement a MESH prototype\(^3\) on top of the GraphX graph processing system built on Apache Spark. Using this prototype and a number of real datasets and algorithms, we experimentally demonstrate that MESH provides the flexibility to make design choices based on data and application characteristics. We also examine the scalability of our system with cluster size, and compare our MESH implementation with the HyperX [13] hypergraph processing system, demonstrating that a simple and flexible implementation need not come at the cost of performance.

Throughout this paper we explore two key research challenges: developing an expressive and easy-to-use API (Section III), and implementing this API on top of an existing graph processing system (Section IV).

II. MESH Overview

A. Design Goals

Expressiveness & Ease of Use: Hypergraph algorithms are fundamentally more general than graph algorithms. Many hypergraph algorithms treat hyperedges as first-class entities on par with vertices. A hypergraph processing system should therefore be expressive enough to allow hyperedges to have attributes and computational functions just as vertices do. It is critical that these attributes and functions be as general for hyperedges as they are for vertices. In addition to this expressiveness, a hypergraph system should also provide ease of use, enabling application developers to easily write a diverse variety of hypergraph applications.

\(^3\)We have released the source code for our implementation (URL not disclosed here due to double-blind requirements).

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\(^2\)Minnesota Engine for Scalable Hypergraph analysis
Scalability: Many real-world datasets range in size from small to massive, comprising millions or billions of vertices and hyperedges. Similar to popular graph processing systems, hypergraph processing systems must be designed to scale to massive inputs, and they must allow distributed processing over multiple machines, while efficiently processing small datasets as well.

Flexibility: A hypergraph processing system must answer two key questions of how to represent hypergraphs, and how to partition this representation for distributed computation. As we show in Section IV, the right answer to these questions depends on many factors related to the input dataset and algorithm characteristics. A hypergraph processing system must therefore be flexible, allowing the appropriate answers for these questions to be made at runtime based on data and application characteristics.

Ease of Implementation: A hypergraph processing system should be designed to simplify implementation as much as practical. This not only allows for faster development with fewer defects, but it allows the system to evolve more rapidly as it gains adoption. This is especially important as hypergraph processing is a novel area, where applications and systems will need to evolve rapidly in tandem.

Existing graph processing systems such as Pregel [2], PowerGraph [15], and GraphX [1] provide the foundation for scalability. They also provide a useful pattern we can follow to achieve programmability, namely the “think like a vertex” programming model, where graph processing applications are expressed in terms of vertex-level programs that iteratively receive messages from their neighbors, update their state, and send message to their neighbors. As we will show, however, these existing systems lack the flexibility required to handle diverse hypergraph applications and data.

B. MESH Hypergraph Processing System

In order to meet the requirements of scalability and ease of implementation, we focus on implementing our hypergraph processing system, called MESH, on top of an existing graph processing system rather than from scratch. We assume that the underlying graph processing framework provides us with a graph representation consisting of vertices and edges, a graph partitioning framework to partition the input data across multiple machines, and a distributed execution framework that supports computation and communication across multiple machines, along with some fault tolerance mechanisms. For our implementation, we choose the GraphX framework [1] in Apache Spark [14]. As Figure 2 shows, MESH is positioned as a middleware layer between hypergraph applications and GraphX.

Given such a system architecture, we explore two key research challenges throughout this paper: developing an expressive and easy-to-use API for enabling diverse hypergraph algorithms (Section III), and implementing this API on top of an existing graph processing system (Section IV).

III. APPLICATION PROGRAMMING INTERFACE

In this section, we first discuss the features of hypergraph algorithms and then present the HPS API that can enable expressing such algorithms easily.

A. Hypergraph Algorithms

Many hypergraph algorithms can be viewed as generalizations of corresponding graph algorithms, but they can have richer attributes and computations, particularly those defined for hyperedges in addition to vertices. We examine some example hypergraph algorithms below to illustrate these aspects.

1) PageRank: Consider PageRank [16], a widely used algorithm in graph analytics to determine the relative importance of different vertices in a graph. It is used in a variety of applications, such as search, link prediction, and recommendation systems.

We can extend PageRank to the hypergraph context in many ways. The most straightforward extension is to compute the PageRank for vertices based on their membership in different hyperedges. In a social context, this would correspond to determining the importance of a user based on her group memberships (e.g., a user might be considered more important if she is part of an exclusive club).

At the same time, it is possible to compute the PageRank for hyperedges based on the vertices they contain. This corresponds to estimating the importance of groups based on their members (e.g., a group with Fortune 500 CEOs is likely to be highly important). This extension also illustrates the fact that hyperedges can be considered first-class entities associated with similar state and computational functions as vertices in typical graph computation.

This elevation of hyperedges to first-class status enables further extensions to PageRank: we can compute additional attributes for hyperedges using arbitrary functions of their member vertices. For example, we can use an entropy function to determine the uniformity of each hyperedge; i.e., the extent to which its members contribute equally to its importance.

2) Label Propagation: Consider a Label Propagation algorithm [13], [17], which determines the community structure of a hypergraph. Here, in addition to identifying the community to which each vertex belongs, we may also assign to each
hyperedge the community to which it belongs. Such an algorithm proceeds by iteratively passing messages from vertices to hyperedges and back. At each step along the way, the solution is refined as vertices and hyperedges update their attributes to record the community to which they belong.

3) Shortest Paths: Consider the Single Source Shortest Paths algorithm that computes the shortest paths from a source vertex to all other vertices in the network. In the hypergraph context, a path would be defined in terms of the hyperedges that are traversed from the source to each destination, and the path length would depend on the number of hyperedges along the path as well as any weights assigned to them. As an example, this can allow us to compute the degree of separation between two users in terms of the group structure of a social network. Conversely, one can also compute shortest paths between hyperedges (e.g., to identify how far two groups are in terms of the connectivity of their users).

Along these same lines, hypergraph extensions can be derived for many popular graph algorithms, such as connected components, centrality estimation [18], and more. The key to this expressiveness is the elevation of hyperedges to first-class status.

B. Core API

To make MESH easy to use, its API builds upon programmers’ existing familiarity with the “think like a vertex” model [2], by providing a “think like a vertex or hyperedge” model. MESH provides an iterative computational model similar to Pregel, but with the introduction of hyperedges as first-class entities with their own computational behavior and state.

In this model, computation proceeds iteratively in a series of alternating “supersteps” (alternating between vertex and hyperedge computation). Within a superstep, vertices (resp., hyperedges) update their state and compute new messages, which are delivered to their incident hyperedges (resp., vertices).

Listing 1 shows the core of the MESH API4. The key abstraction is the HyperGraph, which is parameterized on the vertex and hyperedge attribute data types. Similar to the GraphX Graph interface, the HyperGraph provides methods (not shown) such as vertices and hyperEdges for accessing vertex and hyperedge attributes, mapVertices and mapHyperEdges for transforming the hypergraph, subHyperGraph for computing a subhypergraph based on user-defined predicate functions, and so on.

The iterative computation model described above is implemented via the core computational method, compute. To use the compute method to orchestrate their iterative computation, users encode their vertex (resp., hyperedge) behavior in the form of a Program comprising a Procedure for consuming incoming messages, updating state, and producing outgoing messages, as well as a MessageCombiner for aggregating messages destined to a common hyperedge (resp., vertex). The Context provides methods that enable the Procedure to update vertex (resp., hyperedge) state, and to send messages to neighboring hyperedges (resp., vertices). When a vertex (resp., hyperedge) broadcasts a message, the message is sent to all hyperedges (resp., vertices) to which the vertex (resp., hyperedge) is incident on.

In this model, hyperedges are elevated to first-class status; they can maintain their state, carry out computation, and send messages just as vertices do. The MESH API therefore meets our expressiveness requirements. The generality and conciseness of the API aid in making the API easy to use.

To further improve ease of use, we observe that, in many cases, it is possible to determine the MessageCombiner automatically based on the message types. We implement this convenient feature using Twitter’s Algebird5 library, and allow programmers to enable it with a single import directive. With this feature enabled, users need only specify a Procedure.

C. Example MESH Applications

We next show how we can use the MESH API to implement some of the algorithms discussed in Section III-A. Listing 2 shows the implementation of a hypergraph variant of the PageRank algorithm which computes ranks for both hyperedges and vertices iteratively. As seen from the pseudocode, it is fairly simple to implement the algorithm, requiring only a few lines of code. As shown in Listing 3, a richer version of PageRank which also computes the entropy of each hyperedge (PageRank-Entropy, as described in Section III-A1) requires a simple three-line helper function to compute entropy and changes to only a few other lines (broadcast and rank computation).

4We show Scala code for our API/algorithms. Scala traits are analogous to Java interfaces, and the object keyword here is used to define a module namespace.

5https://github.com/twitter/algebird
Paths, the MessageCombiner (previous iteration). In contrast, every hyperedge and vertex is updated incrementally: if the path length increases, this update is broadcast to neighbors. The algorithm terminates when the updated values they received. The major difference between the Shortest Path algorithm and the other algorithms above is that only a subset of hyperedges and vertices are active during any iteration (ones which were updated with newValue in the previous iteration). In contrast, every hyperedge and vertex is active in every iteration for the other algorithms.

Note that for Label Propagation, PageRank and Shortest Paths, the MessageCombiner is derived automatically.

### Listing 2: PageRank algorithm implementation.

```scala
def entropy(ranks: Seq[Double]): Double = {
  val totalRank = ranks.sum
  val normalizedRanks = ranks.map(_ / totalRank)
  val normalizedRanksSum = normalizedRanks.map {
    p => p * math.log1p(1/p)
  }.sum / math.log2(1)
  ...}
```

### Listing 3: PageRank-Entropy algorithm implementation: Changes from PageRank implementation are shown here.

```
// Attributes have been 'augmented' to carry the state.
type VAtrr = (VD, Rank)
type HEAttreq = (Cardinality, Weight), Rank)
type ToV = (TotalWeight, Rank)
type ToE = Rank

// Vertex procedure: update vertex rank, and broadcast to incident hyperedges.
val prVProc: Procedure[VAtrr, ToV, ToE] = { (ss, id, attr, msg, ctx) => {
  val (totalWeight, rank) = msg
  val (vd, _) = attr
  val newRank = alpha + (1.0 - alpha) * rank
  ctx.become((vd, newRank))
  ctx.broadcast((newRank / totalWeight))
}
```

### Listing 4: Label Propagation algorithm implementation.

```
// Attributes have been 'augmented' to carry the state.
type VAtrr = (VD, Community)
type HEAttreq = (HE, Community)

// Use the same message type in both directions
val Msg = Map[Community, Int]

// Compute the most frequent community
val mostFrequent(msg: Msg): Community = msg.maxBy(_._1)

// Vertex procedure: join the most frequent neighboring community and broadcast this decision.
val lpVProc: Procedure[VAtrr, Msg, Msg] = { (ss, id, attr, msg, ctx) => {
  val (vd, _) = attr
  // To begin, join our own singleton community
  val newCommunity = if (ss == 0) 1d
  else mostFrequent(msg)
  ctx.become((vd, newCommunity))
  ctx.broadcast(Map(newCommunity -> 1))
}
```

### IV. IMPLEMENTATION

In order to implement a scalable hypergraph processing system, we must address two key challenges: how to represent the hypergraph, and how to partition this representation for distributed computation. As discussed in Section II, MESH leverages the capabilities of an underlying graph processing system to address these challenges. Thus, it converts a hypergraph into an underlying graph representation, and utilizes a graph partitioning framework to implement a variety of hypergraph partitioning algorithms. We next discuss the design choices and the tradeoffs in making these decisions, as well as our implementation on top of GraphX.

#### A. Representation

The first question we must address is how to represent a hypergraph as an underlying graph that is understandable by a graph processing system, and we consider two alternatives here.

1) **Clique-Expanded Graph:** One possibility is to represent a hypergraph as a simple graph by expanding each hyperedge into a clique of its members. We refer to this representation as the *clique-expanded graph*. Figure 3(a) shows the clique-expanded representation for the example hypergraph shown in Figure 1(b). In order to enable this representation, our HyperGraph interface provides a toGraph transformation method, which logically replaces the connectivity structure of the hypergraph with edges rather than hyperedges. The attributes of an edge from \(v_1\) to \(v_2\) are determined by user-defined functions applied to the set of all hyperedges common to \(v_1\) and \(v_2\). Applying this transformation to produce a clique-
val augmentedHg = hg.mapHyperVertices(hv =>
  if (hv.id == sourceId) hv.attr -> 0.0
  else hv.attr -> Double.PositiveInfinity)
).mapHyperEdges(he => he.attr -> Double.PositiveInfinity)

val VAttr = (VD, Hops)
val HEAttr = (Cardinality, Weight), Hops
val ToV = (TotalWeight, Hops)
val ToE = Hops

// Vertex Procedure: update vertex shortest hops, and
// broadcast to incident hyperedges when it gets updated.
val prHVProcedure: Procedure[VAttr, ToV, ToE] =
  {ss, id, attr, msg, ctx} => |
    val (vd, value) = attr
    val (totalWeight, update) = msg
    val newValue = update / totalWeight
    if (value > newValue) {
      ctx.become((card, weight), newValue)
      ctx.broadcast(newValue + 1.0)
    }
}

// Hyperedge Procedure: update hyperedge shortest hops,
// and broadcast to incident hyperedges when it gets updated.
val prHEProcedure: Procedure[HEAttr, ToE, ToV] =
  {ss, id, attr, msg, ctx} => |
    val ((card, weight), value) = attr
    val newValue = msg
    if (value > newValue) {
      ctx.become((card, weight), newValue)
      ctx.broadcast((weight, newValue))
    }
}

// Initial message sent to all vertices
val initialMessage = (0.0, Double.PositiveInfinity)

Listing 5: Shortest Path algorithm implementation.

Fig. 3. Underlying graph representations of the hypergraph in Figure 1(b).

expanded graph may be costly—even prohibitively so—in terms of both space and time.

Another major disadvantage of the clique-expanded graph
is its limited applicability. Because hyperedges do not appear in this representation, it is only appropriate for algorithms
that do not modify hyperedge state, and thus, for instance cannot be used for our Label Propagation algorithm. Further, the
hyperedge and vertex programs must meet additional
requirements, such as sending the same message type in both
directions. Overall, therefore, this representation is best viewed as a potential optimization for a small set of use cases rather
than a general approach.

2) Bipartite Graph: An alternative approach is to represent
the hypergraph internally as a bipartite graph, where one partition comprises exclusively vertices, and the other exclusively
hyperedges, with low-level graph edges connecting hyperedges
to their constituent vertices. Figure 3(b) shows the bipartite
representation for the example hypergraph from Figure 1(b).
This representation can concisely encode any hypergraph, and
it allows us to run programs that treat both vertices and
hyperedges as first-class computational entities. By using
directed edges (in our implementation exclusively from vertices
to hyperedges), we provide a means to differentiate between vertices and hyperedges. Due to the general expressive power
of this representation, we focus our attention throughout this paper on its efficient implementation in a graph processing
system.

B. Partitioning

1) Challenges: To scale to large hypergraphs, it is essential
to distribute computation across multiple nodes. The decision
of how to partition the underlying representation can significantly affect performance, in terms of both computational load and network I/O. An effective partitioning algorithm—
whether for a graph or a hypergraph—must simultaneously balance computational load and minimize communication.
Hypergraph partitioning, however, presents several challenges beyond those for partitioning graphs.

For one, hypergraphs contain two distinct sets of entities:
vertices and hyperedges. In general, these two sets can differ significantly in terms of their size, skew in cardinality/degree6,
and associated computation. Further, MESH computation runs on only one of these sets at a time. An effective partitioning
algorithm must therefore differentiate between hyperedges and vertices.

At the same time, hyperedge and vertex partitioning are
fundamentally interrelated; an effective algorithm must holistically partition hyperedges and vertices. For example, an
algorithm that partitions hyperedges without regard to vertex
partitioning may achieve good computational load balance, but will suffer from excessive network I/O.

2) Algorithms: MESH utilizes the underlying graph partitioning framework to implement hypergraph partitioning
algorithms. Many graph processing frameworks either partition vertices (cutting edges7) or partition edges (thus cutting
vertices) across machines. Many current systems [1], [15]
use edge partitioning since it has been shown to be more efficient for many real-world graphs. In what follows, we
describe mapping hypergraph partitioning algorithms to such
dge partitioning graph algorithms. We expect that mapping
to vertex partitioning algorithms could be done in a similar
fashion, and we leave such mapping as future work.

Concretely, we assume the underlying graph partitioning framework partitions the set of edges, while replicating each vertex
to every partition that contains edges incident on that vertex. In our bipartite graph representation, edges are directed
exclusively from (hypergraph) vertices to hyperedges. As a

6The degree of a vertex denotes the number of hyperedges of which that vertex is a member. Similarly, the cardinality of a hyperedge denotes
the number of vertices belonging to that hyperedge.

7Note that we use “edge” to refer to an edge in the underlying graph representation, and it is not to be confused with a hyperedge in the provided
hypergraph.
result, if we partition based only on the source (resp., destination) of an edge, hypergraph vertices (resp., hyperedges) are each assigned to a unique partition, while hyperedges (resp., vertices) will be replicated—i.e., “cut”—across several partitions. If we choose the partition for an edge based on both its source and destination, then both vertices and hyperedges are effectively cut.

Any graph partitioning algorithm leads to a tradeoff between balancing computational load and minimizing network communication. While balancing the number of edges across machines could lead to good load balance, a high degree of replication of vertices can lead to increased network I/O and execution time due to increased syncing and state updates. In order to distribute a hypergraph, however, replication is unavoidable. The goal is therefore to choose which set(s) (vertices or hyperedges) to cut, and how to partition the other set so as to balance computational load while minimizing replication.

We explore a range of alternative partitioning algorithms that approach this goal from different angles. These algorithms fall into three classes: Random, Greedy, and Hybrid. We illustrate each of these algorithms by showing how they would partition our example hypergraph bipartite representation (Figure 3(b)) on two machines.

a) Random: We explore three Random partitioning algorithms. The Random Vertex-cut algorithm hash-partitions bipartite graph edges based on their destination (i.e., by hyperedge), effectively cutting hypergraph vertices. For example, in Figure 4(a), the algorithm assigns each hyperedge to either machine1 or machine2 using a hash function. It then assigns a replica of each vertex to every machine which contains its incident hyperedges. E.g.: \(v_1\) is assigned to machines 1 and 2, as it is incident on \(he_1\) and \(he_2\) (on machine 1), and \(he_3\) (on machine 2). The Random Hyperedge-cut algorithm, on the other hand, partitions hyperedges and cuts vertices, as shown in Figure 4(b).

The Random Both-cut algorithm hash-partitions bipartite graph edges by both their source and destination, effectively cutting both vertices and hyperedges.

b) Hybrid: The Hybrid algorithms we consider are based on the balanced \(\rho\)-way hybrid cut from PowerLyra [19]. These algorithms cut both vertices and hyperedges, but unlike Random Both-cut, they differentiate between vertices and hyperedges in doing so. The Hybrid Vertex-cut variant cuts vertices while partitioning hyperedges, except that it also cuts hyperedges with high cardinality (greater than 100 in our experiments). In Figure 5(a), the algorithm cuts vertices \(v_1\) and \(v_3\) while partitioning hyperedges and cuts hyperedge \(he_2\) since it has cardinality greater than the cutoff value of 3 in the example. Similarly, the Hybrid Hyperedge-cut variant cuts hyperedges while also cutting high-degree vertices, as shown in Figure 5(b).

c) Greedy: Based on the Aweto [20] algorithm, the Greedy algorithms that we consider holistically partition hypergraphs with the goal of reducing replication and the resulting synchronization overhead. At a high level, the Greedy Vertex-cut variant aims to assign each hyperedge to a lightly-loaded partition with a large “overlap” between the vertices in
that hyperedge and the vertices with replicas already on that partition based on (a heuristic estimate of) the assignments already made. (For a more rigorous definition, see [20].)

Figure 6 illustrates the details of Greedy partitioning strategies, showing how the strategies incrementally assign vertices and hyperedges. In Figure 6(a), the Greedy Vertex-cut algorithm first hash-partitions the bipartite graph edges based on their vertices and then assigns one hyperedge at a time to an appropriate machine. \( T = t_1 \) in Figure 6(a) shows the intermediate state after hyperedges \( h e_1 \) and \( h e_2 \) have been assigned. Hyperedge \( h e_1 \) is assigned to machine 1 because the machine contains maximum number of incident vertices at this time. Because load and overlap are even across machines at this time, hyperedge \( h e_2 \) is randomly assigned to machine 1, and \( v_3 \) and \( v_4 \) are cut accordingly. \( T = t_2 \) in the figure shows the final state of the partitioning. Hyperedges \( h e_3 \) and \( h e_4 \) are assigned to machine 2 because it contains maximum overlapping edges and the machine is also lightly loaded at this time.

The Greedy Hyperedge-cut variant in Figure 6(b) works similarly, except it assigns vertices based on the overlap between their incident hyperedges and the hyperedges already assigned to each partition.

C. Implementation on GraphX

As mentioned in Section II, we have implemented a MESH prototype on top of the GraphX [1] graph processing system. GraphX provides a graph representation consisting of a VertexRDD and an EdgeRDD which internally extend Spark’s Resilient Distributed Datasets (RDDs), an immutable and partitioned collection of elements [14]. VertexRDD contains information about vertex ids and vertex attributes. EdgeRDD contains information about edges (pairs of vertices) and edge attribute properties.

In our implementation, Hypergraph contains an additional HyperEdgeRDD which is similar to VertexRDD and contains information about hyperedge ids and hyperedge attributes. Moreover, EdgeRDD now contains information about (vertex, hyperedge) pairs and the attribute properties for the relation between them. We can represent a hypergraph using a clique representation by mapping each hyperedge to a clique of its incident vertices. Similarly, we can represent a hypergraph as a bipartite graph by creating edges between vertices and their hyperedges.

GraphX uses an edge-partitioning algorithm for partitioning the graph across machines. In GraphX, the PartitionStrategy considers each edge in isolation, as in Listing 6.

```scala
def getPartition(
    src: VertexId,
    dst: VertexId,
    numPart: PartitionId): PartitionId
```

Listing 6: Original GraphX partitioning abstraction.

We use the built-in GraphX partitioning algorithms to implement the baseline Random partitioning algorithms described above. Our Greedy and Hybrid algorithms, however, require a broader view of the graph (to compute “overlap” and degree/cardinality, respectively). To satisfy this requirement, we extend the PartitionStrategy by adding a new getAllPartitions method that allows partitioning decisions to be made with awareness of the full graph, as shown in Listing 7.

Unlike the getPartition method of GraphX, which receives source and destination vertices for an edge and returns partition number for that edge, the getAllPartitions method receives property graph corresponding to a pair of VertexRDD and EdgeRDD and returns an RDD which maps source and destination vertices with their associated partition number as shown in Listing 7. Next, we discuss how Hybrid and Greedy partitioning algorithms use this extended partitioning interface.

```scala
def getAllPartitions[VD, ED](
    graph: Graph[VD, ED],
    numPartitions: PartitionId,
    degreeCutoff: Int): RDD[((VertexId, VertexId), PartitionId)]
```

Listing 7: Extended GraphX partitioning abstraction.

Hybrid Vertex-cut PartitionStrategy uses different partitioning policy for low and high degree vertices. In this PartitionStrategy, if the cardinality of a particular hyperedge exceeds the provided threshold (degreeCutoff), it cuts the hyperedge and partitions it based on the hashing of source vertex; otherwise, it partitions based on the hashing of destination vertex as shown in Listing 8. It is done to reduce communication overhead due to high degree hyperedges.

Listing 9 shows Greedy Vertex-cut PartitionStrategy which uses overlap and load for partitioning. This PartitionStrategy considers one hyperedge at a time and greedily chooses a partition that the hypervertices contained in this hyperedge most overlapped with. Additionally, if the load on the chosen partition is high, it picks the next most overlapped partition.

```scala
Listing 8: Hybrid Vertex-cut PartitionStrategy.
```

V. EVALUATION

A. Experimental Setup

1) Deployment: We implement and run our MESH prototype on top of Apache Spark 1.6.0. Experiments are conducted on a cluster of eight nodes, each with two Intel Xeon E5-2620 v3 processors with 6 physical cores and hyperthreading...
### Table I

**DataSets Used in Our Experiments.**

<table>
<thead>
<tr>
<th>Dataset</th>
<th># Vertices</th>
<th># Hyperedges</th>
<th>Max. Degree</th>
<th>Max. Cardinality</th>
<th># Bipartite Edges</th>
<th># Clique-Expanded Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apache</td>
<td>3316</td>
<td>78,080</td>
<td>4,507</td>
<td>179</td>
<td>408,231</td>
<td>196,452</td>
</tr>
<tr>
<td>dblp</td>
<td>899,393</td>
<td>782,659</td>
<td>368</td>
<td>2,803</td>
<td>2,624,912</td>
<td>21,707,061</td>
</tr>
<tr>
<td>Friendster</td>
<td>7,944,949</td>
<td>1,620,991</td>
<td>1,700</td>
<td>9,299</td>
<td>23,479,217</td>
<td>10.3 billion (approximate)</td>
</tr>
<tr>
<td>Orkut</td>
<td>2,322,299</td>
<td>15,301,901</td>
<td>2,958</td>
<td>9,120</td>
<td>107,080,530</td>
<td>54.5 billion (approximate)</td>
</tr>
</tbody>
</table>

---

```scala
// Count the number of edges corresponding to each DstId
val groupedDst = graph.edges.map{ e => (e.dstId, e.srcId, e.attr)}.groupByKey

// Count the overlap for each DstId with each Partition.
val dstOverlap = new Array[Long](numPartitions)
  e._2.map{ src =>
    dstOverlap(math.abs(src * mixingPrime) % numPartitions)}
  dstOverlap.map{ e =>
    e._1, e._2).groupByKey

// Iterate each partition to find most overlapping partitions among neighbors of this vertex.
val mostOverlap = Double.MaxValue
  cur = numPartitions
  for (part = 0 until numPartitions-1)
    val overlap = Double.MinValue
    cur = math.sqrt(1.0*current_load(cur))
    if (overlap > mostOverlap)
      mostOverlap = overlap
      cur = cur
  cur = cur
  for (part = 0 until numPartitions-1)
    if (part = cur)
      mostOverlap = overlap
      cur = cur

// Increase the edge count of this partition
for (part = 0 until numPartitions-1)
  if (part = cur)
    dstOverlap.map{ e =>
      e._1, e._2).groupByKey

```

---

Fig. 7. Run time comparison for bipartite and clique-expanded representations for the PageRank algorithms on the Apache and dblp datasets. Friendster and Orkut results are not shown since their clique-expansions could not be materialized.

---

The Apache hypergraph, derived from the Apache Software Foundation subversion [22] logs, models collaboration on open-source software projects. Each vertex represents a committer, and each hyperedge represents a set of committers that have collaborated on one or more files.

The dblp dataset describes more than one million publications, from which we use authorship information to build a hypergraph model where vertices represent authors and hyperedges represent collaborations between authors.

In the Friendster and Orkut hypergraphs, vertices represent individual users, and hyperedges represent user-defined communities in the Friendster and Orkut social networking sites, respectively. Because membership in these communities does not require the same commitment as collaborating on software or academic research, these hypergraphs have very different characteristics from dblp and Apache, in particular in terms of the overall size of the data, and vertex degree and hyperedge cardinality. One difference between the two is that Friendster has many more vertices than hyperedges, whereas the opposite is true for Orkut.

3) Applications: We use the four applications described in Section III-C in our experiments: the Label Propagation algorithm (Listing 4), the two PageRank variants: PageRank (Listing 2) and PageRank-Entropy (Listing 3), and the Shortest Paths algorithm (Listing 5).

---

B. Representation

We first briefly explore the relative strengths and weaknesses of two main representation alternatives: the clique-expanded graph and the bipartite graph. Table I shows the number of edges required for each of these representations, while Figure 7 shows both partitioning time and subsequent execution time for the PageRank algorithm for the Apache and dblp hypergraphs. Throughout this paper, we run each experiment multiple times and plot the mean, with error bars denoting 95% confidence intervals.

For the Apache hypergraph, the clique-expanded graph shows some promise in terms of both space and time. The clique-expanded representation uses only about 48% as many edges as the bipartite alternative, and although the initial par-
titioning phase (which includes running the toGraph transformation) is more time-consuming for this representation, the execution is significantly faster. For the dblp hypergraph, the clique-expanded representation again shows an execution time advantage, but this comes at the cost of space overhead, as this representation requires roughly 8x as many edges as the bipartite alternative.

The clique-expansion can be thought of as a constant-folding optimization applied at the time of constructing the representation. Although this can be helpful in terms of execution time in some cases, its space overhead can be large or even prohibitive. In fact, for the Friendster and Orkut hypergraphs, we are unable to even materialize the clique-expanded graphs on our cluster due to space limitations. As seen from Table I, these datasets would result in approx. 10 billion and 54 billion clique-expanded edges respectively, which is orders of magnitude higher than that for their bipartite graph representation.

In addition to these scalability concerns, it is important to keep in mind that the clique-expanded representation does not apply for all algorithms, as discussed in Section IV. For example, we cannot use this representation for our Label Propagation or PageRank-Entropy algorithms, as these algorithms need explicit access to hyperedge attributes. Thus, while the clique-expansion representation might be beneficial for some use cases (specific algorithms and small datasets), it is neither expressive enough nor scalable in general. Given these limitations, we focus on the more general alternative of the bipartite representation throughout the remainder of our experiments.

C. Partitioning

Next, we evaluate the partitioning policies described in Section IV. Due to space constraints, we omit results for the Apache dataset here. Figure 8 shows both partitioning time and subsequent execution time for the Label Propagation algorithm for each of these policies for the dblp, Friendster, and Orkut datasets. Figures 9, 10, and 11 repeat these experiments for the PageRank, PageRank-Entropy, and Shortest Paths algorithms respectively.

We see that the choice of the best partitioning algorithm depends on the data. One possible data characteristic having an impact could be the relative number of vertices and hyperedges in the hypergraph. First considering Figures 8 (Label Propagation) and 9 (PageRank), we see that the greedy hyperedge-cut algorithm is the best for the Friendster hypergraph (Figures 8(b) and 9(b)), where vertices outnumber hyperedges. Here, cutting hyperedges while partitioning the larger set of vertices might lead to better computational load balancing. On the other hand, for the Orkut hypergraph (Figures 8(c) and 9(c)), where hyperedges outnumber vertices, we see that while the vertex-cut algorithms seem to perform better than the corresponding hyperedge-cut variants, a Random Both-cut algorithm is the best. This suggests that cutting vertices is better than cutting hyperedges, but that cutting both sets may lead to even better load balancing. For dblp (Figures 8(a) and 9(a)), we see a much less pronounced difference between vertex-cut and hyperedge-cut algorithms, as the number of hyperedges and vertices in this dataset are roughly equal.

Figures 10 and 11 shows the results for the PageRank-Entropy and Shortest Path algorithms, and their trends are very similar to those of the PageRank and Label Propagation algorithms, including the best partitioning strategy for each dataset. Note that the execution times of Shortest Path algorithm shown in Figure 11 are smaller than those of other algorithms because it terminates when messages are passed through the maximum distance between any two vertices, i.e., the diameter of the graph, whereas the other algorithms run more iterations until the values of vertices and hyperedges are converged or exceed the maximum number of iterations (30 for our experiments).

These results show that no one partitioning algorithm dominates all others in all cases. The best choice depends on the characteristics of the hypergraph. For instance, holistically partitioning the hypergraph, as done by the Greedy vertex-cut and hyperedge-cut algorithms, can be beneficial in some cases, while cutting both hyperedges and vertices can be effective in others. A promising next step is to develop a combined algorithm that partitions holistically as the Greedy algorithms do, while differentiating between hyperedges and vertices as the Random Both-cut and Hybrid algorithms do.

These results also show the value of the flexibility provided by MESH, where the choice of an appropriate partitioning algorithm can be based on data and application characteristics. Note that the vertex-to-hyperedge ratio is only one data characteristic that may be impacting the performance. Identifying all the relevant characteristics and their impact, and automatically making the design choices is an area of future work.

D. Scaling

Our next set of experiments examine the scaling of MESH based on available computing resources (size of the cluster). For each dataset, we show results for strong scaling, i.e., keeping the total dataset size the same, while changing the number of nodes in the cluster. We ran the Label Propagation algorithm for all partitioning strategies and for all datasets described above with 2, 4, 6, and 8 nodes, and measured the partitioning and execution times. For each cluster size, the input dataset was reloaded into HDFS restricted to the given set of cluster nodes.

Figure 12 shows the scaling results for the four datasets using the Random Both-cut partitioning strategy and Figure 13 shows detailed results for all partitioning strategies (Apache is omitted due to space constraints). We make the following observations from these results. First, as seen from the figures, the execution times for all datasets either decreases or flattens out as the size of the cluster increases. The bigger the size of the dataset is, the greater the performance benefit. For instance, in Figure 13(c), for Orkut, the execution time keeps decreasing as the computing resources increase from 2 to 8 nodes, indicating a computational resource bottleneck at smaller cluster sizes. However, if the computing resources are sufficient to deal with a given dataset (e.g., Apache and dblp),
Fig. 8. Label Propagation: Partitioning and execution time using several partitioning algorithms in MESH.

Fig. 9. PageRank: Partitioning and execution time using several partitioning algorithms in MESH.

Fig. 10. PageRank-Entropy: Partitioning and execution time using several partitioning algorithms in MESH.

Fig. 11. Shortest Paths: Partitioning and execution time using several partitioning algorithms in MESH.
the performance improvement obtained by running on a larger cluster becomes insignificant. For example, Figure 13(a) shows that for dblp, there is only a slight performance improvement when the size of the cluster increases from two to four nodes for all partition strategies. This means that a 2-node cluster is sufficient for the dblp dataset.

Figure 12 also shows that, although the Friendster and Orkut datasets have different input data sizes, their execution times are saturated around 3000 seconds with 8 nodes. Orkut dataset contains two times more the total numbers of vertices and hyperedges and 4.5 times more bipartite edges than Friendster dataset, but the maximum cardinality of Orkut is similar to that of Friendster (about 9,000 from Table I). Once the cluster is large enough that computational resources are sufficient, the network I/O becomes a bottleneck. Since the messages sent from one host to another are merged before they are sent out, the maximum cardinality influences the message sizes and hence, the total network traffic, leading to a similar performance for both Orkut and Friendster at larger cluster sizes.

E. Comparison with HyperX

To evaluate the overall performance of MESH, we compare against HyperX [13], a hypergraph processing system that is also built on top of Apache Spark. Unlike MESH, which builds on top of GraphX, HyperX implements a hypergraph layer—heavily inspired by GraphX—directly on top of Spark. While the HyperX implementation is optimized for hypergraph execution, our implementation relies on the GraphX optimizations designed for graph execution. Here, we evaluate the performance tradeoff given the simplicity of our API and implementation. We compare these two systems using a Label Propagation algorithm, specifically Listing 4 for MESH, and the provided example implementation for HyperX.8

Figure 14 shows the partitioning and Label Propagation execution times (for 30 iterations) for HyperX and MESH (using the best partitioning policies for the given dataset).

Unlike MESH, HyperX uses an iterative partitioning algorithm (10 iterations in our experiments, based on the HyperX experiments [13]), leading to much higher partitioning times. This costly partitioning may be warranted for the dblp, Friendster and Orkut hypergraphs, as HyperX achieves lower execution time than MESH for those hypergraphs. For Apache, however, MESH achieves lower application runtime in spite of its less sophisticated partitioning algorithm.

In terms of performance, these results show the efficacy of MESH, which achieves comparable performance to HyperX, despite lacking several low-level optimizations. An additional qualitative benefit of MESH is its flexibility: hypergraphs are diverse, and MESH provides a simple interface that allows implementing different partitioning policies easily, as shown above.

From a higher level, our results suggest that high performance need not be at odds with a simple and flexible implementation. In fact, by layering on top of GraphX and leveraging its maturity and ongoing development, we can expect to reap the benefits of ongoing optimization. Backporting future optimizations to HyperX, on the other hand, would require significant engineering effort.

VI. RELATED WORK

a) Graph Processing Systems: There has been a flurry of research on graph computing systems in recent years [2], [15], [23], and along with it, a great deal of work on performance evaluation and optimization [24]–[26].

Key among these systems, Pregel [2] introduced the “think like a vertex” model. GraphX [1], built upon Apache Spark [14], adopted a similar model while inheriting the scalability and fault tolerance of Spark’s Resilient Distributed Datasets (RDD). GraphLab [27] provided a more fine-grained interface along with support for asynchronous computation.

These systems provide scalability, and their interfaces are easy to use in the graph computing context. Our MESH API can be viewed as an extension of the “think like a vertex” model. Although we have discussed challenges in implementing MESH on top of a graph processing system in general, and GraphX in particular, there is no fundamental requirement that MESH run on top of a specialized platform. For example, MESH could be implemented on top of a general-purpose relational database management system (RDBMS) [28]. GraphX, however, is particularly compelling due to the popularity and growth of Spark. Further, by facilitating diverse views of the same underlying data—e.g., collection-oriented, graph-oriented, tabular [29]—building on top of Spark allows easier integration in broader data processing pipelines.

b) Graph and Hypergraph Partitioning: Graph Partitioning is a significant research topic in its own right. In the high-performance computing context, metis [30] provides very effective graph partitioning, and has open-source implementations for both single-node and distributed deployment. Its hMetis [30] cousin partitions hypergraphs, but no distributed implementation yet exists. The Zoltan toolkit from Sandia
National Laboratories [31] includes a parallel hypergraph partitioner [32] that cuts both vertices and hyperedges.

In the distributed systems context, PowerGraph [15] targets natural (e.g., social) graphs by cutting vertices rather than edges. While this is effective for natural graphs, hypergraphs require different approaches. Chen et al. have proposed novel algorithms for bipartite graphs [1] and skewed graphs [19], which we have used as the basis for our Greedy and Hybrid algorithms, respectively. While these are already effective algorithms, there remains opportunity to combine holistic and differentiated approaches to improve hypergraph partitioning.

c) Hypergraph Processing: Hypergraphs have been studied for decades [5], [6] and have been applied in many settings, ranging from bioinformatics [10] to VLSI design [33] to database optimization [12]. Social networks have generally been modeled using simple graphs, but hypergraph variants of popular graph algorithms (e.g., centrality estimation [8], [18], shortest paths [9]) have been developed in recent years. HyperX [13] builds a hypergraph processing system on top of Spark, but does so by modifying GraphX rather than building on top of GraphX. Unlike HyperX, MESH does not make any static assumptions about the data characteristics, and instead provides the flexibility necessary to choose an appropriate representation and partitioning algorithm at runtime based on data and application characteristics.

VII. CONCLUSION

We presented MESH, a flexible distributed framework for scalable hypergraph processing based on a graph processing system. MESH provides an easy-to-use and expressive API that naturally extends the “think like a vertex” model common to many popular graph processing systems. We used our system to explore two key challenges in implementing a hypergraph processing system on top of a graph processing system: how to represent the hypergraph and how to partition this representation to allow distributed computation. MESH provides flexibility to implement different design choices, and by implementing MESH on top of the popular GraphX framework, we have leveraged the maturity and ongoing development of the Spark ecosystem and kept our implementation simple. Our experiments with multiple real-world datasets and algorithms demonstrated that this flexibility does not come at the expense of performance, as even our unoptimized prototype performs comparably to HyperX.

REFERENCES


